Frequency Analysis of Articulated Robot

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Abstract

This paper presents a comprehensive frequency analysis of articulated robot. The purpose of a frequency response analysis is to compute the behavior of a articulated robot subjected to time-varying excitation. The transient excitation is explicitly defined in the time domain. The force applied to the structure is known at each instant in time. Forces can be in the form of applied forces or enforced motions. The important results obtained from the frequency analysis are typically displacements, velocities, accelerations, eigenvalues, eigenvectors, and mode shapes of the robot nodes. Depending upon the structure and the nature of the loading, two different methods are used for the frequency response analysis direct and modal.The direct method performs an analytical analysis on the complete coupled equations of motion. The modal method utilizes the mode shapes of the robot to reduce and uncouple the equations of motion (when modal method is used); the solution is then obtained through the summation of the individual modal responses.

تحليل الترددات لإنسان آلي من النوع المفصلي

الخلاصة

يتضمن هذا البحث دراسة موسعة لتحليل الترددات الخاصة بالإنسان الآلي(الروبوت) من النوع المفصلي . الغرض من الدراسة هو لتحليل استجابة الإنسان الآلي المعرض لإثارة متغيرة مع الزمن ، فالإثارة العابرة والقوة المسلطة على هيكل الإنسان الآلي معروفة انيا في مجال الزمن حيث يمكن أن تكون القوة أما على شكل قوى مسلطة مباشرة أو حركات قسرية فقط .من النتائج المهمة التي تم الحصول عليها من تحليل التردد للإنسان الآلي هي الإزاحات، السرع،التسارع(التعجيل)، الترددات الطبيعية، المتجهات والنسوق في العقد العائدة للإنسان الآلي . تم استخدام طريقتين مختلفتين لتحليل الاستجابة للتردد اعتمادا على هيكل الإنسان الآلي وطبيعة الحمل وهي الطريقة المباشرة وطريقة النسوق، الطريقة المباشرة تم فيها انجاز تحليل كامل لمعادلات الحركة المزدوجة بينما في طريقة النسوق فتم الاستفادة من الأشكال النسقية الظاهرة للإنسان الآلي لتقليل وعدم ازدواجية المعادلات حيث تم الحصول على الحل بتجميع الاستجابات النسقية المتراكمة.

NOMENCLATURE

A = Cross sectional area, [mm^2]
a_R, a_θ, a_β = Robot acceleration,[mm/sec^2]
C = Damping matrix, [N.s/m]
Cc_r= Critical damping, [N.s/m]
$D_r(t)$ = Dynamic load factor
E_i = Modulus of elasticity for link i, [N/m^2]
I_i = Area moment of inertia for link i, [m^4]
K = Stiffness matrix, [N/m]
L = Robot link length, [mm]
M = Mass , [Kg]
m_i = Mass for link i,[kg]
m_s = Cross sectional mass of the beam, [Kg]
n = Constant

P = Force applied, [N]
Q_i = The ith generalized external force, [N]
$Q(t)$ = External force, [N]
q_i = Normal coordinate,[mm]
R = The dissipation energy, [Joule]
T = The kinetic energy, [Joule]
t = Time, [sec]
U = The strain energy, [Joule]
$\upsilon_R, \upsilon_\theta, \upsilon_\beta$ = Robot velocities ,[mm/sec]
$y(x,t)$ = Motions coordinate, [mm]
$\phi_i(x)$ = The transverse deflection along the ith link,[m]
ω_i = Natural frequency for link i,[rad/sec]
$\phi_{ij}(x)$ = Mode shape function
ρ = Density, [kg/m^3]
ξ = Damping ratio
ω_d = Damping frequency,[rad/sec]
τ_d = Time delay,[sec]
$H(i\omega)$ = Complex frequency response
ω_n = Natural frequency, [rad/sec]
q_{stat} = Static modal deflection,[mm]
f_i = Frequency,[Hz]
σ = Response stress, [N /m^2]
σ_a = Stress amplitude,[N /m^2]
R_p = Radius, [mm]
β, α, γ = Robot angles,[deg.]
λ = Distance between the action force and support,[mm]

Introduction

The concept of industrial robot was first patented in 1954 by (G.C Devol) who described how to construct a controlled mechanical arm, which can perform industrial tasks. The first industrial robot was installed by (unimation) Inc. in 1961 and since that thousands of robots have been put to work in industry in U.S, Japan, and Europe, to perform a wide range of tasks such as loading and unloading , spot and arc welding, spray painting, die casting, forging, inspection, plastic molding , and glass industry, material handling, space technology, and assembly tasks [1],[2].

The robot is defined by the American institutes of robots, as a programmable multi-functional manipulator designed to move materials, parts, tools, or special devices, through a variable programmed motion for the performance of a variety of tasks [3],[4].

Two approaches used in controlling robots the first is torque control by manipulating the motor current, and is mainly used by the USA robot manufacture. The second is speed control by manipulating the motor applied voltage and is mainly followed by Japanese and European manufacture [5].

The frequency analysis of articulated robot [6], shown in Fig. (1) **is a subject of great importance and it has a wide application of robot design and control. For high-speed** robot application, the frequency, characteristics of the robot must be taking into account to achieve smooth motion and stable control. The study of the dynamic behavior of flexible small and medium size robots with joint compliance is the bases for developing efficient and robust controllers for such system [7].

The paper consider the vibration of articulated robot consist of three rigid members (links) connected by two revolute joint on a rotary base. Rotary and linear motions are introduced to the fixed end and at the joint, which couples the three links. Inertias are lumped at the coupling joints and at the free end of the last link. The Lagrange equation for a beam is used for modeling the vibration of robot uniform elastic links with the suitable conditions.

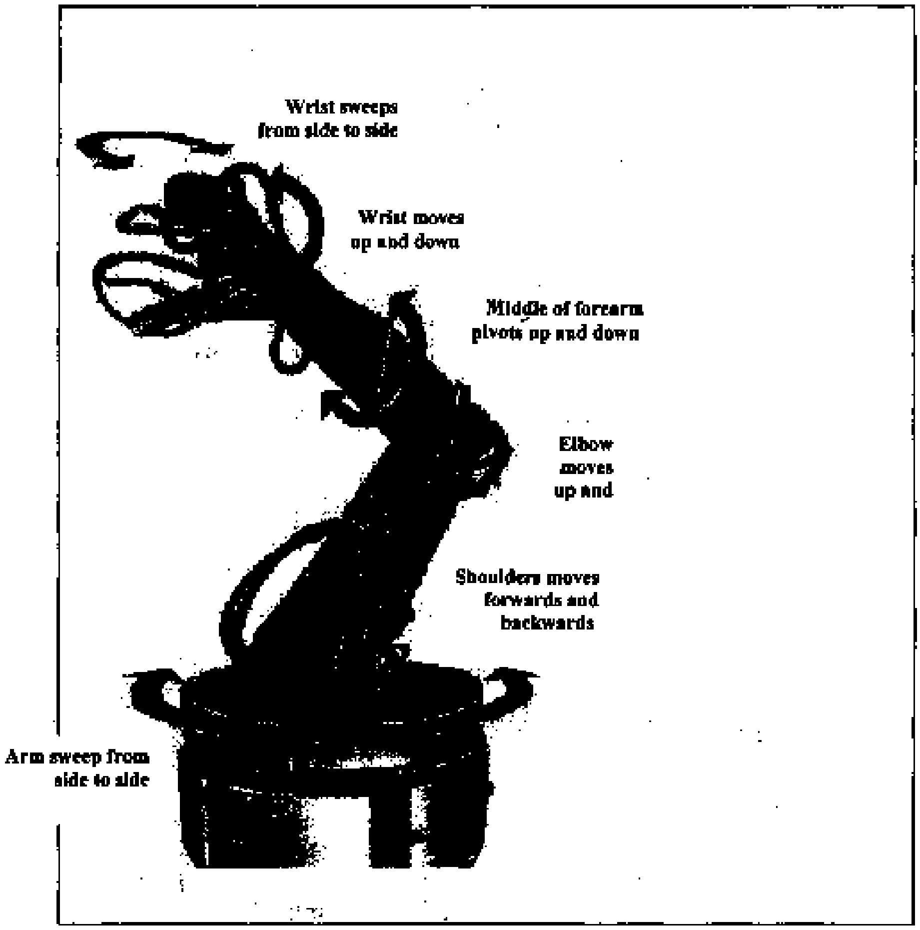

Fig.(1) Articulated robot

Characteristic equation

The normal mode method is characterized by the fact that the differential equations of motion are decoupled when the displacements are expressed in terms of the normal modes [9].Therefore ,in a system having n degrees of freedom ,we may deal with n independent differential equations rather than with a system of n simultaneous differential equations.

The Lagrange equation [10] for a beam with n independent differential equations is given by:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right)-\frac{\partial T}{\partial q_i}+\frac{\partial U}{\partial q_i}+\frac{\partial R}{\partial \dot{q}_i}=Q_i \quad (1)$$

Where T is the kinetic energy and U is the strain energy.
R is the dissipation energy and Q_i denotes the ith generalized external force in their normal coordinate system *and* q_i is the normal coordinate.

The response of the Articulated robot model shown in Fig.(1) can be defined by the generalized normal coordinates

$$y(x,t)=\sum_{i=1}^{n}\phi_{ij}(x)q_i(t) \quad (2)$$

Where $\phi_{ij}(x)$ is the mode shape function.
For the slender beam the kinetic and the strain energy are:

$$T=\frac{1}{2}\int_o^l m_i \dot{y}^2\, dx \quad (3)$$

$$U=\frac{1}{2}\int_o^l E_i I_i y'' dx \quad (4)$$

$$R=\frac{1}{2}\int_o^l C_i \dot{y}^2\, dx \quad (5)$$

In which m_i, E_i, I_i, C_i is the mass ,modulus of elasticity, area moment of inertia, and the damping per unit length of beam respectively, the integrals of the Eqs.(3),(4)and(5) are taken over the length of the beam.

From equation (2) we can write

$$\dot{y}(x,t)=\sum_{i=1}^{n}\phi_{ij}(x)\dot{q}_i,\ \dot{y}=dy/dt \quad (6)$$

$$y'(x,t)=\sum_{i=1}^{n}\phi'_{ij}(x)q_i,\ y'=dy/dx \quad (7)$$

Substituting these equations into Eqs. (3), (4) and (5) and determine the partial Derivatives as:

$$\left.\begin{aligned}
&\frac{\partial T}{\partial \dot{q}_i}=\dot{q}\sum_{i=1}^{n}\int_0^l \rho A\phi_{ij}^2 dx+m_s\phi_s^2\\
&\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right)=\ddot{q}\sum_{i=1}^{n}\int_0^l \rho A\phi_{ij}^2 dx\\
&\frac{\partial U}{\partial q_i}=q_i\sum_{i=1}^{n}\int_0^l E_i I_i \phi_{ij}''^2 dx+K_i\phi_{ij}\\
&\frac{\partial R}{\partial \dot{q}_i}=\dot{q}_i\sum_{i=1}^{n}\int_0^l c_i\phi^2{}_{ij} dx\\
&Q_o=\sum_{i=1}^{n}Q_i\phi_{ij}
\end{aligned}\right\} \quad (8)$$

Where m = ρA , per unit length and ρ, A , m_s , Ki are the density, Cross sectional area , Cross sectional mass , and Stiffness matrix of the beam, respectively.

Put equation (8) into Lagrange equation

$$\ddot{q}\sum_{i=1}^{n}\int_{0}^{l}\rho A\phi_{ij}^{2}dx+q_{i}\sum_{i=1}^{n}\int_{0}^{l}E_{i}I_{i}\phi_{ij}''^{2}dx+\dot{q}_{i}\sum_{i=1}^{n}\int_{0}^{l}c_{i}\phi^{2}ij\,dx=\sum_{i=1}^{n}Q_{i}\phi_{ij} \qquad (9)$$

By Comparison with the generalized equation of motion

$$M\ddot{q}+C\dot{q}+Kq=Q_{i}(t) \qquad (10)$$

and the Generalized mass

$$m_{ij}=\int_{0}^{l}\rho A\phi_{ij}^{2}dx+m_{s}\phi_{s}^{2} \qquad (11)$$

also the Generalized stiffness coefficient is

$$K_{ij}=\int_{0}^{l}E_{i}I_{i}\phi_{ij}''^{2}dx+K_{i}\phi_{i}^{2} \qquad (12)$$

Where the first term is for beams and the second for springs Generalized damping coefficient

$$C_{i}=\int_{0}^{l}c_{i}\phi^{2}ij\,dx \qquad (13)$$

$$\left.\begin{aligned} C_i &= 2\zeta_i m_{ij}\omega_i \\ \zeta_i &= \frac{c_i}{Cc_n} \\ Cc_n &= 2m_{ij}\omega_i \\ \omega d_i &= \sqrt{1-\zeta_i^2}\,\omega_i \\ \tau_d &= \frac{2\pi}{\omega d_i} \end{aligned}\right\} \quad (14)$$

The Generalized force

$$Q(t) = Q_i(t)\phi_{ij} \qquad (15)$$

Letting the capital letters stand for the Fourier transforms of the corresponding quantities in lower case letters.

The complex frequency response is obtained by making the substitution:

$$\left.\begin{aligned} x_q(t) &= \int_{-\infty}^{\infty} X_q(i\omega)e^{i\omega t}d\omega \\ x_Q(t) &= \int_{-\infty}^{\infty} X_Q(i\omega)e^{i\omega t}d\omega \\ \dot{x}_q(t) &= \int_{-\infty}^{\infty} i\omega X_q(i\omega)e^{i\omega t}d\omega \\ \ddot{x}_Q(t) &= \int_{-\infty}^{\infty} -\omega^2 X_Q(i\omega)e^{i\omega t}d\omega \end{aligned}\right\} \quad (16)$$

in Equation (10) and canceling the $e^{i\omega t}$ terms yields

$$H(i\omega) = \frac{\text{output}}{\text{input}} = \frac{X_q(i\omega)}{X_Q(i\omega)} = \left[K - \omega^2 M + i\omega C\right]^{-1}$$

(17)

Where H(iω) is the complex frequency response function for the output y(t) due to external force (input) Q(t).

The square modulus of the complex frequency response is:

$$|H(\omega)|^2 = \frac{1}{K^2[(1-\omega^2/\omega_n^2)+(2\zeta\omega/\omega_n)^2]}$$

(18)

Where ζ, ω_n damping factor, natural frequency

$$q_i(t) = Q_i(t) * h(t) \qquad (19)$$

where * = convolution h(t)
H(jω)

Static modal deflection

By making the substitution of Eq. (10) and rearranging we obtain the forms of the static modal deflection $q_{istat}(t)$:

$$\left.\begin{aligned} q_{j\,stat}(t) &= \sum_{i=1}^{m} \frac{Q_i(t)\pi f_i \phi_{ij}^2}{4\zeta_i K_i^2} \\ q_{j\,stat}(t) &= \frac{\sum_{i=1}^{m} P_i \phi_{ij}}{\sum_{i=1}^{m} \omega_i^2 m_i \phi_{ij}} \\ q_{j\,stat}(t) &= \sum_{i=1}^{m} \frac{k_i^2\left(1+\zeta_i^2\right) Q_i(t)\phi_{ij}^2}{4\zeta_i \omega_i^3} \\ q_{j\,stat}(t) &= \sum_{i=1}^{m} \frac{Q_i(t)\pi \phi_{ij}^2}{2k_i c_i} \end{aligned}\right\}$$

(20)

Where the time is:

$t_i = 2\pi/\omega_i$ sec (21)

and the frequency is,

$$f_i = \frac{\omega_i}{2\pi} = \frac{1}{T_i} \quad \text{Hz} \quad (22)$$

The spherical coordinates of the articulated robot

$$v = \dot{R}p e_R + Rp\dot{\beta} e_\beta + Rp\dot{\theta}\cos\beta e_\theta \quad (23)$$

Where the space velocities are:

$$\left.\begin{aligned} v_R &= \dot{R}\,p \\ v_\beta &= Rp\,\dot{\beta} \\ v_\theta &= Rp\,\dot{\theta}\cos\beta \end{aligned}\right\} \quad (24)$$

and the accelerations are:

$$\left.\begin{aligned} a_R &= \ddot{R}p - Rp\dot{\beta}^2 - Rp\dot{\theta}^2\cos^2\beta \\ a_\theta &= Rp\ddot{\beta}\cos\beta + 2\dot{R}p\dot{\theta}\cos\beta - 2\dot{R}p\dot{\theta}\sin\beta \\ a\beta &= Rp\ddot{\beta} + 2\dot{R}\dot{\beta} + Rp\dot{\theta}^2\sin\beta\cos\beta \end{aligned}\right\}$$

(25)

Modal Frequency response analysis

For simply-supported beam [11] the mode shape equation is

$$\phi(x) = \sin\frac{n\pi x}{L} \quad (26)$$

Total dynamic deflection of concentrated (point) load is by substitution the above Equations. In the Equation:

$$y_i(x,t) = \frac{2Q(t)}{mL} \sum_{n=1}^{k} \left\{ \frac{1}{\omega_n^2} \left[\sin\frac{n\pi}{L} - \sin\frac{n(L-\lambda)}{L} \right] \left(D_r(t)\right)_n \sin\frac{n\pi x}{L} + q_{j\,stat}(t) \right\}$$

.... (27)

Where $D_r(t)$ is called the dynamic load factor . It is a dimensionless quantity [12] which is a function of time it is depend upon the force time function q(t) ,upon the natural frequencies of the structure in the rth mode ω_r ,and upon the damping coefficient C . λ is the distance between the action force and the support.

The total deflection at any point is obtained by superimposing the modes

$$y_i(x,t) = \sum_{i=1}^{n} q_i(t)\phi_{ij}\left(D_r(t)\right)_i \quad (28)$$

and the modal matrix for 3-mode shape is:

$$\left[\phi_{ij}\right]=\begin{bmatrix}\phi_{11} & \phi_{12} & \phi_{13}\\ \phi_{21} & \phi_{22} & \phi_{23}\\ \phi_{31} & \phi_{32} & \phi_{33}\end{bmatrix}$$

$, i, j = 1,2,3$

The matrices [m] and [k] can be obtained by Rayleigh-Rits method which leads to evaluate eigenvalues and eigenvectors [13]:

$$\left.\begin{aligned} m_{ij} &= \int_o^l m(x)\phi_i(x)\phi_j(x)dx \\ k_{ij} &= \int_o^l EA(x)\phi_i'(x)\phi_j'(x)dx \end{aligned}\right\} \quad (30)$$

$$\det\left|[K]-\omega_i[M]\right|\phi_i = 0 \quad (31)$$

Normal mode function is determined by means of a procedure as The Rayleigh-Ritz or Galerkin [14] method thus;

$$\phi_r(x)=\sum_{i=1}^{n}\phi_i(x)q_i^{(r)} \quad (32)$$

Where;

$\phi_r(x)$ is the mode shape function

$q_i^{(r)}$ is the modal column

and the ith link natural frequency (eigenvalues) is

$$\omega_i^{\ 2}=\frac{[\phi_{ij}]^T[K][\phi_{ij}]}{[\phi_{ij}]^T[M][\phi_{ij}]} \quad (33)$$

Where [K] and [M] denote for stiffness and mass matrix

To find eigenvectors (29)

$$\phi_{ij}=\frac{[\text{adj}K]}{\det[K]}P_j \quad (34)$$

or

$$\phi_{ij}=\frac{[K^C]^T}{|K|}P_j \quad (35)$$

or

$$\phi_{ij}=\frac{k_{ij}}{|K|}P_j \quad (36)$$

Where P_j is the single applied force and k_{ij} is the element of the matrix [adjK] in the ith row and jth column

For beam of circular cross sectional area the moment of inertia is

$$I=\pi d^4/64 \quad (37)$$

Response stress [15]:

$$\sigma = ky = \frac{kA}{\sqrt{2}}\sin(\omega_o t-\theta) \quad (38)$$

where ω_o is the natural frequency

Stress amplitude

$$\sigma_a=\frac{kA}{\sqrt{2}} \quad (39)$$

Results and Discussion

The frequency response of the articulated robot is computed for a range of values of exciting frequencies. The exciting load is a force Q(t) applied at the free end at an angle γ with respect to the axis of the third link as shown in Fig.(2). Table (1) describes the model articulated robot specification used in this work [16],[17].

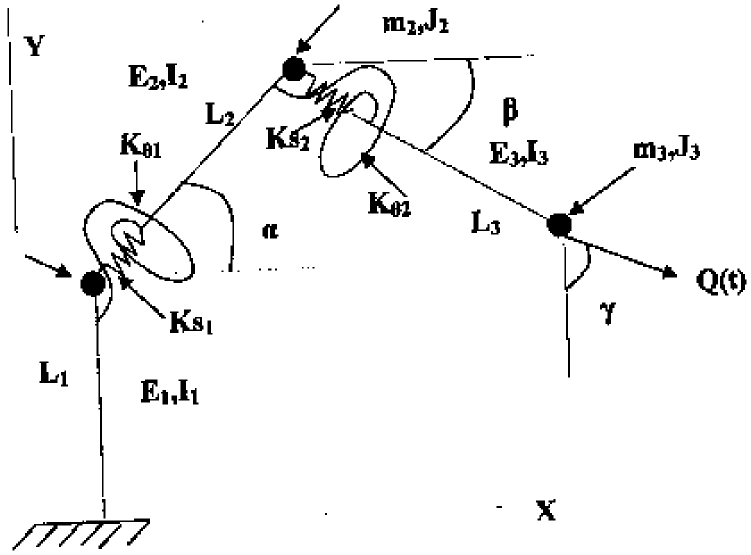

Fig. (2) Articulated robot with all parameters shown (Robot geometry)

Table (1) Articulated robot Specifications

Item		Units	Specifications
Degree of freedom			4 axes
Structure			multi-articulation type
Driving method			AC servo motor
Position sensing method			Absolute encoder
Maximum load capacity (rating)		kg	15 (5)
Arm length	Arm No. 1	mm	525
	Arm No. 2	mm	325
Maximum reach radius		mm	850
Working area	J1 axis(1st axis)	Deg.	±140
	J2 axis(2nd axis)	Deg.	±153
	J3 axis (Z axis)	mm	350
	J4 axis (O axis)	Deg.	±360
Maximum speed	J1 axis	Deg./s	252
	J2 axis	Deg./s	337.5
	J3 axis (Z axis)	mm/s	1000
	J4 axis (O axis)	Deg./s	1274
Horizontally-resultant maximum speed		mm/s	5650
Cycle time		sec	0.57
Allowable wrist-moment of inertia (rating)		kgm^2	0.2 (0.02)
Repeatability	X-Y composite	mm	±0.025
	J3 axis (Z axis)	mm	±0.01
	J4 axis (O axis)	Deg.	±0.03
Ambient temperature		°C	0 to 40
Weight		kg	40
Operating pressure		MPa Air	0.05 to 0.35

Fig.(3) shows the geometry building of the robot using ANSYS 5.4 program, which indicates the entire system of the robot that included all parameters taking into account in this study. The deflected and undeflected shape are shown in Fig.(4) result from the solution of the program and the mode shape for the first three natural frequency are shown in Fig.(5) which depending on the computational accuracy.

To obtain more accuracy results used to analyze the frequency of the articulated robot which are useful to obtain the eigenvalues and eigenvectors used in the model of this study to compute the accumulated deflection in the last link that effect on the resolution of the robot ,more than 10th mode shapes is analyzed as shown in Figures(6-12). Fig. (13) Shows the vector notation of the articulated robot which indicates the deflection in every node of the robot during the motion of the work volume of the robot. The accumulated deflection during the natural frequencies can be obtain from the deformation of the robot and animation so that can be used in the model without need to solving the equation of motion and this can be seen in the figures (14-18). Figures (19-24) indicates the time history where the resonance frequencies of the flexible joint system are low, the exciting frequency range is much larger than that for the stiff joint system.

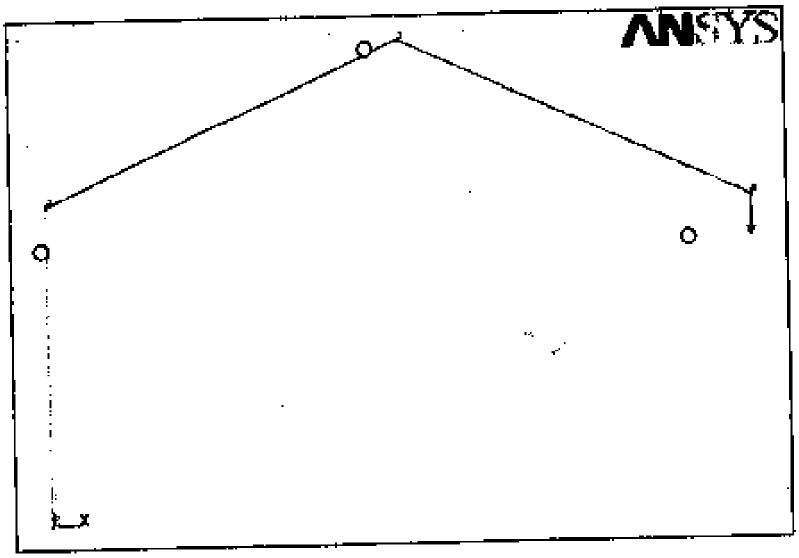

Fig. (3) Robot geometry

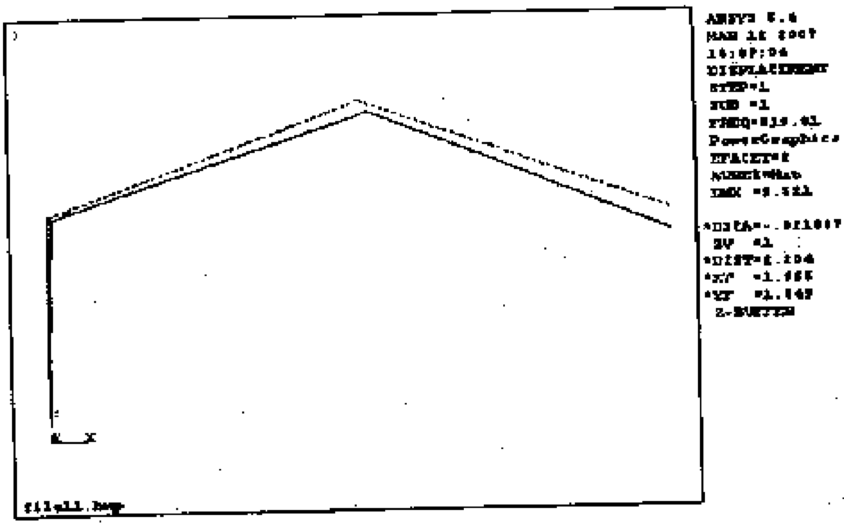

Fig. (4) Deflected and Undeflected shape

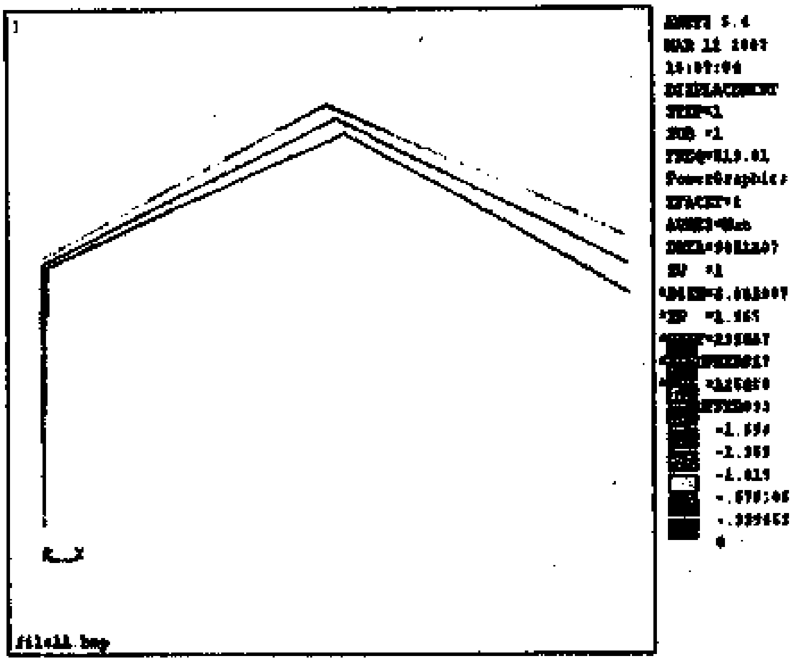

Fig. (5) The mode shape for the first three natural frequencies

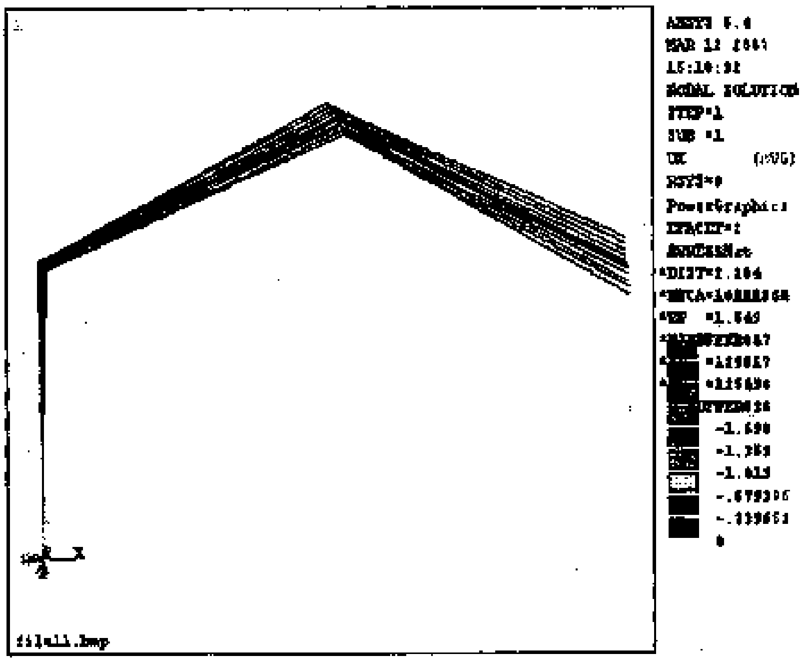

Fig. (6) Mode shapes

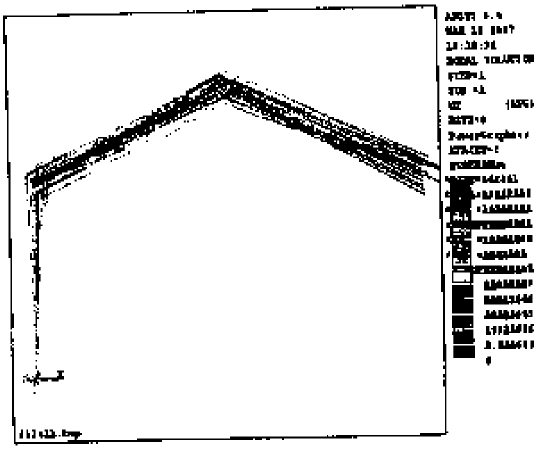

Fig. (7) 1st Eigenvalue Mode shapes

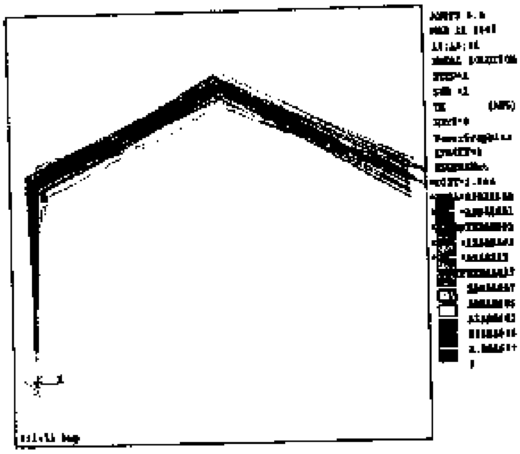

Fig. (8) 2nd Eigenvalue Mode shapes

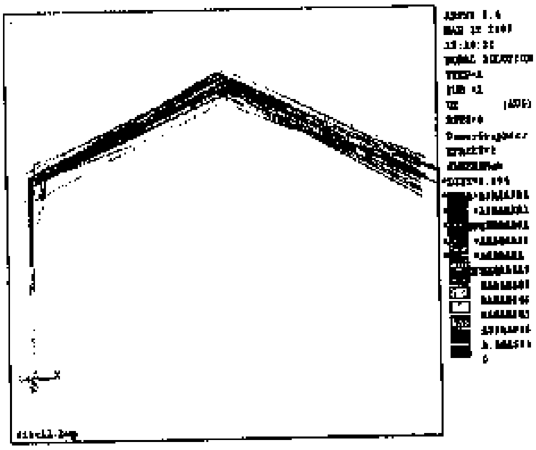

Fig. (9) 3rd Eigenvalue Mode shapes

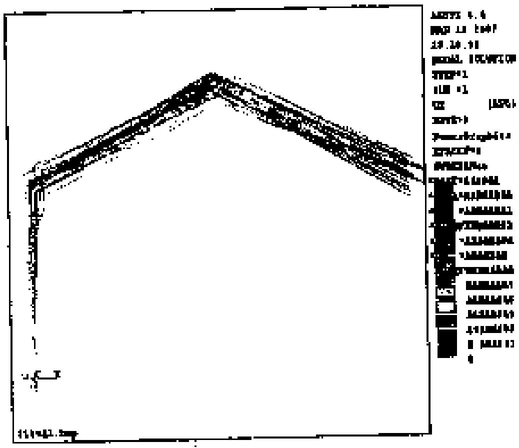

Fig. (10) 4th Eigenvalue Mode shapes

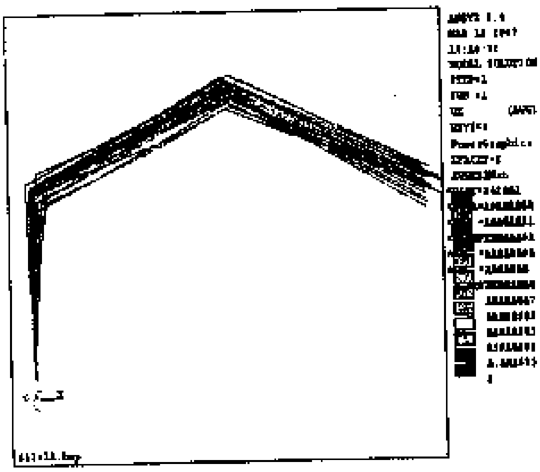

Fig. (11) 5th Eigenvalue Mode shapes

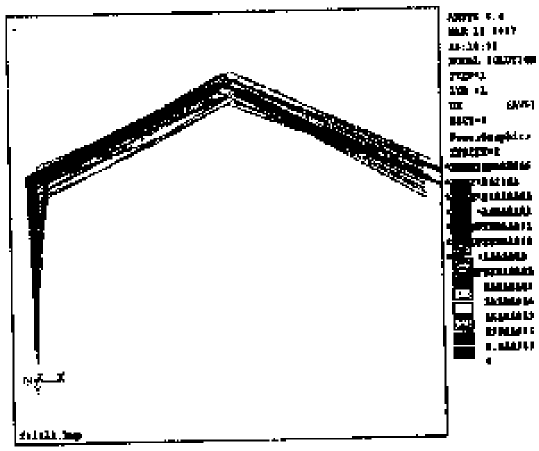

Fig. (12) 6th Eigenvalue Mode shapes

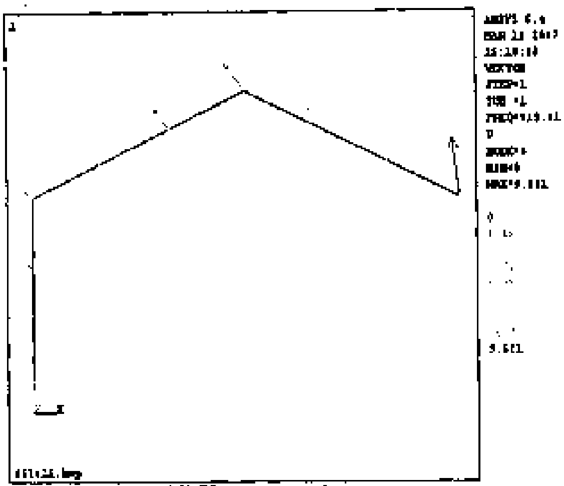

Fig. (13) Vector notation

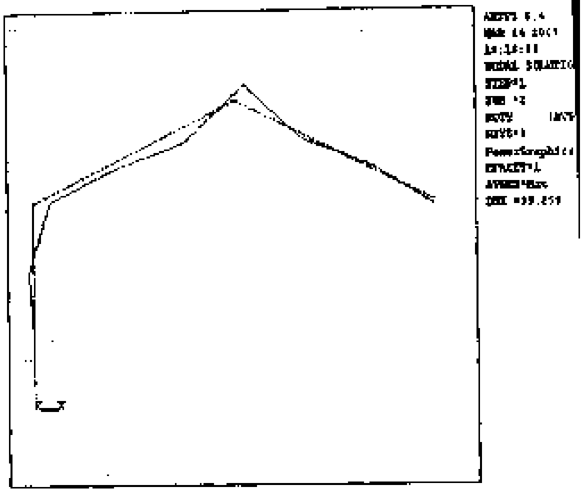

Fig. (14) Deformation and non deformation of the articulated robot

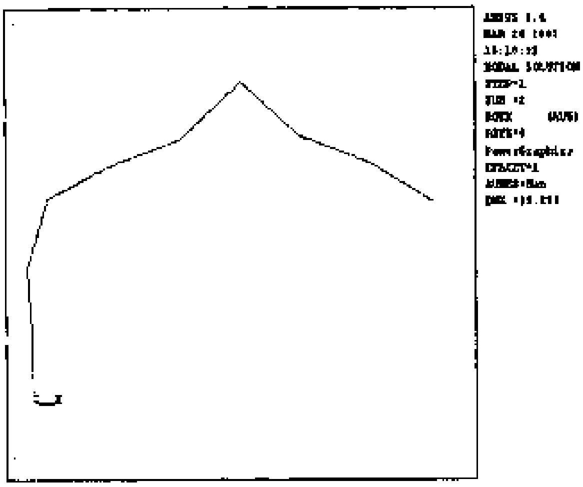

Fig. (15) Robot deformation (X Rotation)

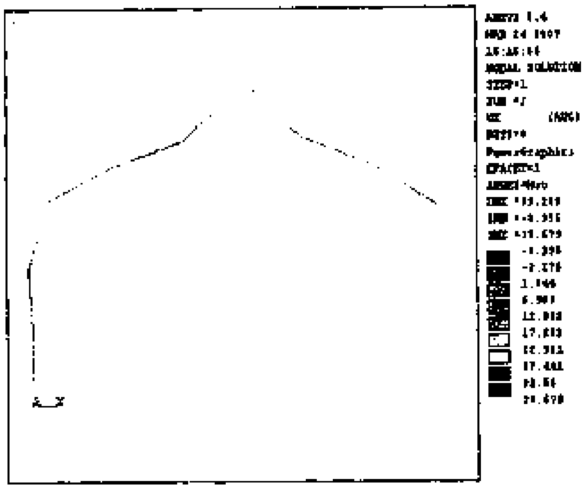

Fig. (16) Robot deformation (Y Rotation)

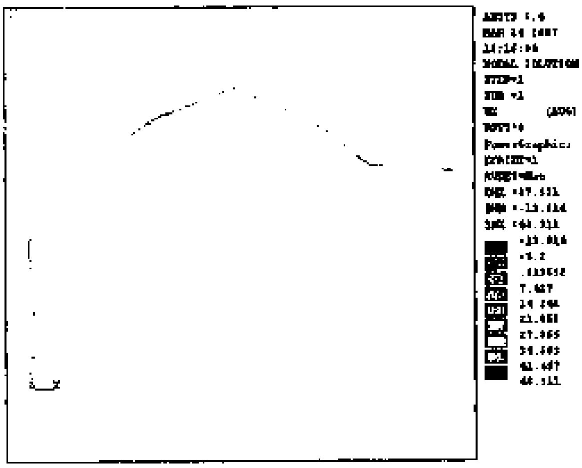

Fig. (17) Robot deformation (Z Rotation)

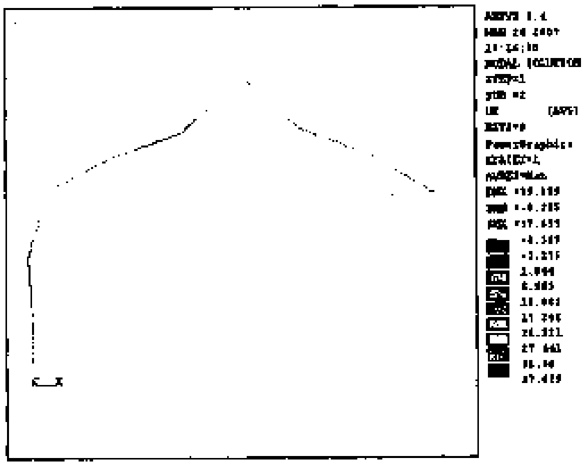

Fig. (18) Robot deformation (Translation)

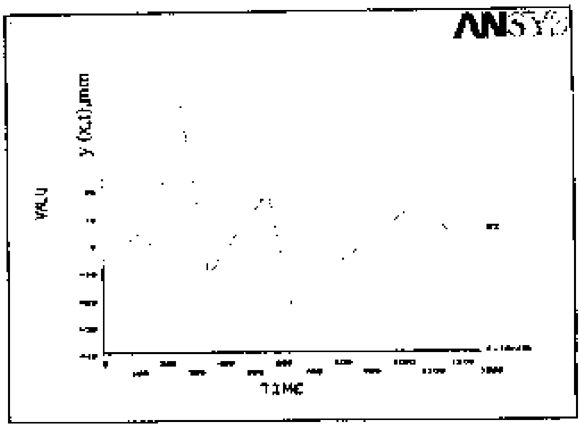

Fig. (19) Time history (1st mode)

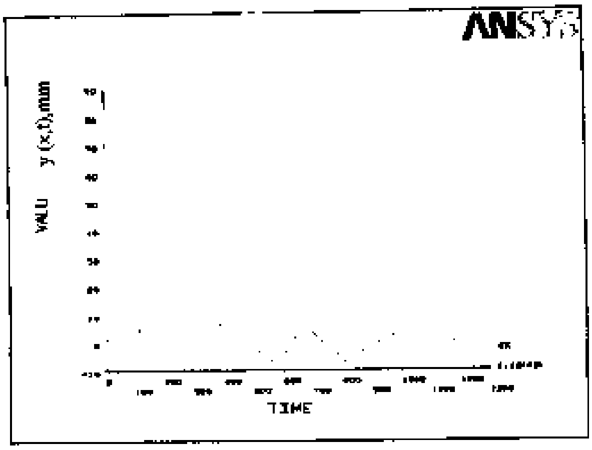

Fig. (20) Time history (2nd mode)

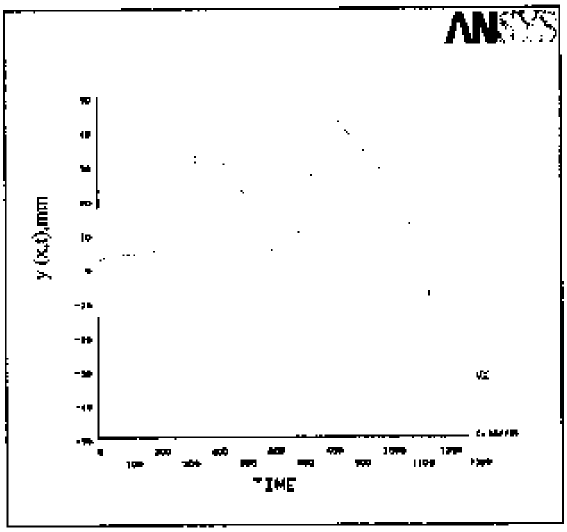

Fig. (21) Time history (3rd mode)

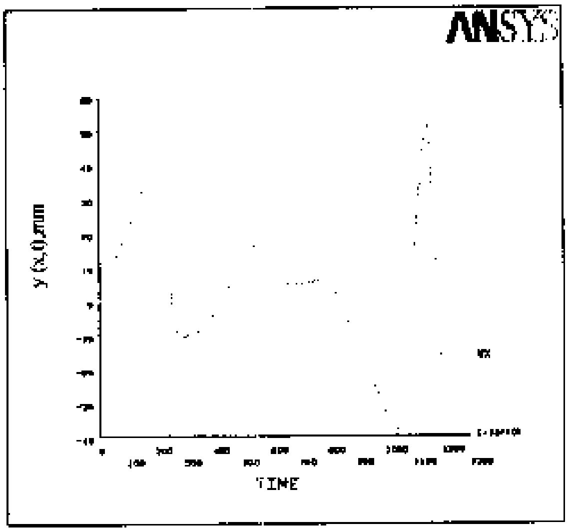

Fig. (22) Time history (4th mode)

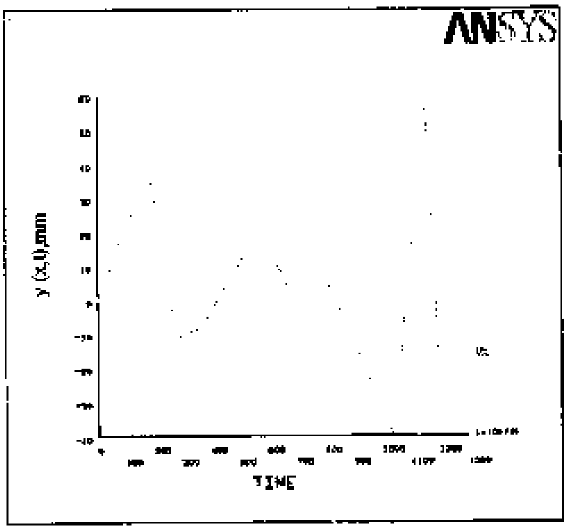

Fig. (23) Time history (5th mode)

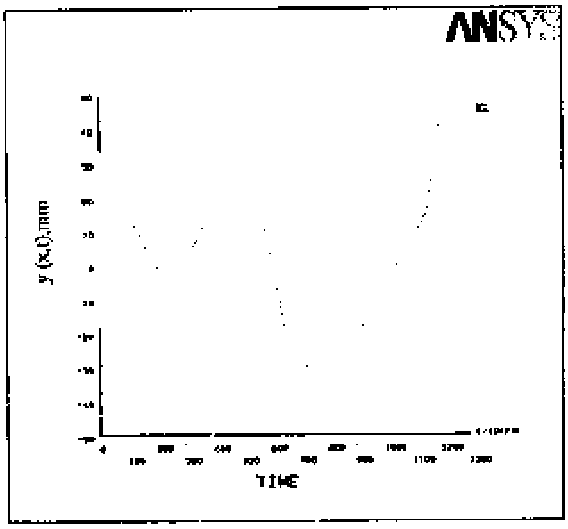

Fig. (24) Time history (6th mode)

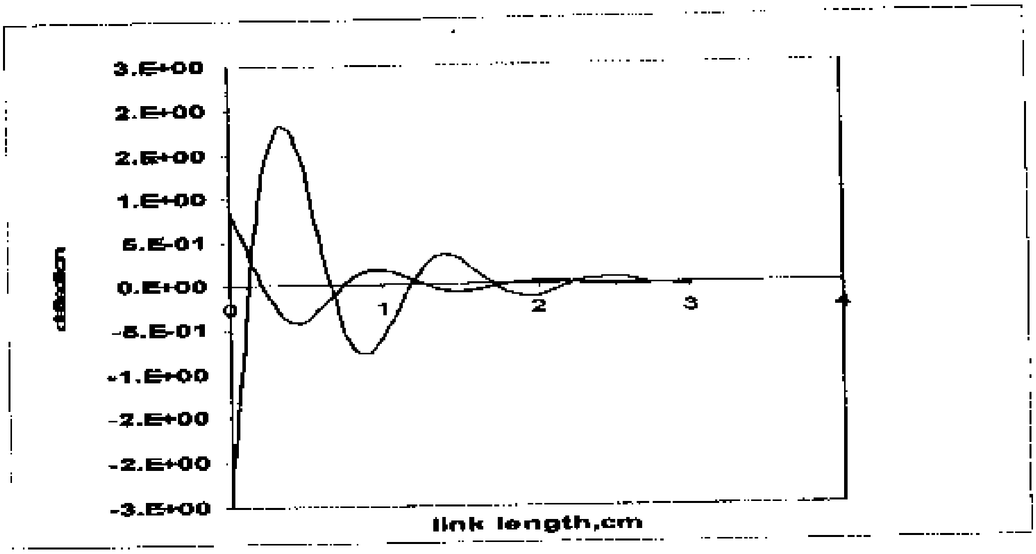

Fig. (25) The mode shapes of the first tow modes of vibration

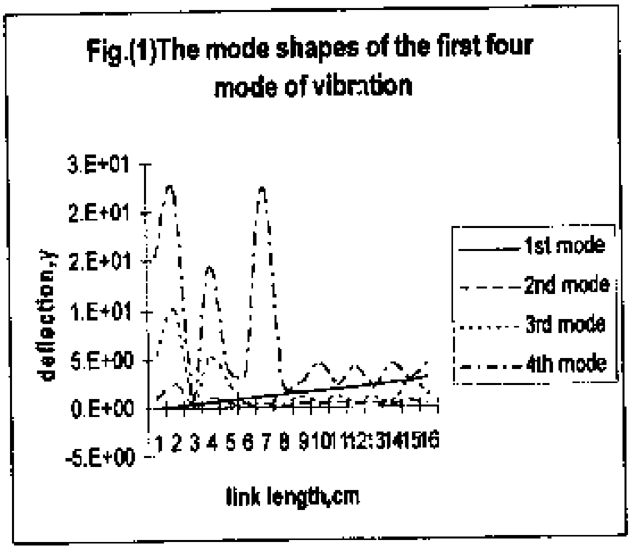

Fig. (26) The mode shapes of the first four mode of vibration

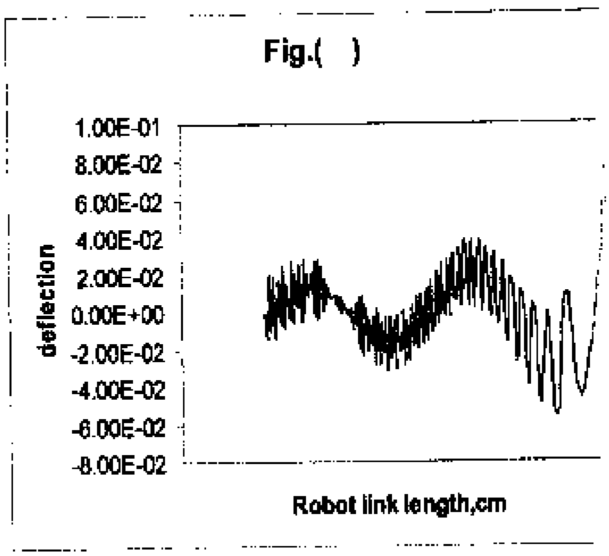

Fig. (27) Rapid mode shape due rapid motion of ro

Analytical analysis

In direct transient response, structural response is computed by solving a set of coupled equations using direct numerical integration. The fundamental structural response (displacement) is solved at discrete times, typically with a fixed integration time step. A central finite difference representation for the velocity and acceleration is used at discrete times, and the applied force is averaged overthrew adjacent time steps. Figures (25, 26) shows that the first two resonance frequencies of the flexible joint system are low; the exciting frequency range between the second and third resonance frequencies is much larger than that for the stiff joint system. This is an advantage of having a flexible joint in the three-link system giving it the ability to be operated at wide range of load excitation. The disadvantage of the rapid motion caused rapid mode shape, which becomes near the resonance region and this undesirable work volume [see Fig.(27)].

Conclusion

The modal analysis of an articulated robot was performed .The natural frequencies and mode shapes were obtained for several parameters combination. Frequency response analysis was performed by finding the vibrational amplitudes and the accumulated deflections in the free end of the robot.

Dynamic Analysis to analyze articulated robot subject to loads that vary with time or frequency. All links of the robot have resonant or natural frequencies, and if the structure is excited at, or close to one of these frequencies then a very high amplitude response can occur. Therefore, it is necessary to ensure in the design of the articulated robot that the resonant and excitation frequencies are not close to each other.

The behavior of a structure to time-varying excitation is computed. Frequency response analysis computes the structural response to steady-state oscillatory excitation. In addition, it is also possible to conduct a random analysis with frequency response.

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